Equazioni e disequazioni con il valore assoluto

$$|2x^2-1|-2=x$$

$$|2x^2-1|=x+2$$

$$|a(x)|=b(x)$$

$$\begin{cases} a(x)\geq 0 \\ a(x)=b(x) \end{cases} \vee \begin{cases} a(x)<0 \\ -a(x)=b(x) \end{cases}$$

$$① \begin{cases} 2x^2-1\geq 0 \\ 2x^2-1=x+2 \end{cases} \vee ② \begin{cases} 2x^2-1<0 \\ -2x^2+1=x+2 \end{cases}$$

①

$$2x^2-1=x+2$$

$$2x^2-x-2-1=0$$

$$2x^2-x-3=0$$

$$x_{1,2}=\frac{1\pm\sqrt{1+24}}{4}=\frac{1\pm 5}{4}=\begin{cases}\frac{6}{4}=\frac{3}{2} \\ \frac{-4}{4}=-1\end{cases}$$

$\underline{x = \frac{3}{2}}$ $\boxed{2x^2 - 1 \geq 0}$

$2 \cdot \left(\frac{3}{2}\right)^2 - 1 = \cancel{2} \cdot \frac{9}{\cancel{4}_2} - 1 =$

$= \frac{9}{2} - 1 > 0$ $\left(\frac{3}{2}\right.$ È ACCETTABILE)

$x = -1$

$2 \cdot (-1)^2 - 1 \geq 0$

$2 \cdot 1 - 1 \geq 0$

$1 \geq 0$ vera ($x = -1$ È ACCETTABILE)

Passiamo a risolvere il secondo "sistema"

$1 - 2x^2 = x + 2$

$2x^2 + x + 2 - 1 = 0 \Rightarrow 2x^2 + x + 1 = 0$

$2x^2+x+1=0$ non ammette soluzioni reali perché

$\Delta = 1-8 = -7 < 0$

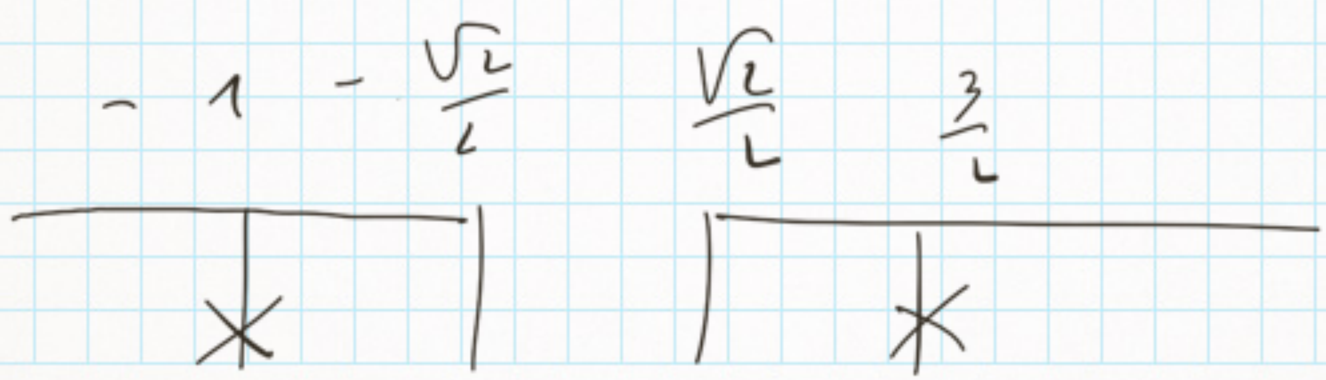
Allora le soluzioni di

$$|2x^2-1| = x+2$$

sono $x = \frac{3}{2}$ e $x = -1$

Si può usare anche il metodo "grafico"

$$\begin{cases} 2x^2-1 \geq 0 \Rightarrow x \leq -\frac{\sqrt{2}}{2} \vee x \geq \frac{\sqrt{2}}{2} \\ x = \frac{3}{2} \vee x = -1 \end{cases}$$



Risolviamo la disequazione

$$|2x^2-1| \geq x+2$$

$$\begin{cases} 2x^2-1 \geq 0 \\ 2x^2-1 \geq x+2 \end{cases} \vee \begin{cases} 2x^2-1 < 0 \\ 1-2x^2 \geq x+2 \end{cases}$$

$$\begin{cases} 2x^2-1 \geq 0 \\ 2x^2-x-3 \geq 0 \end{cases} \vee \begin{cases} 2x^2-1 < 0 \\ \underbrace{2x^2+x}_{\Delta<0}+1 \leq 0 \end{cases}$$

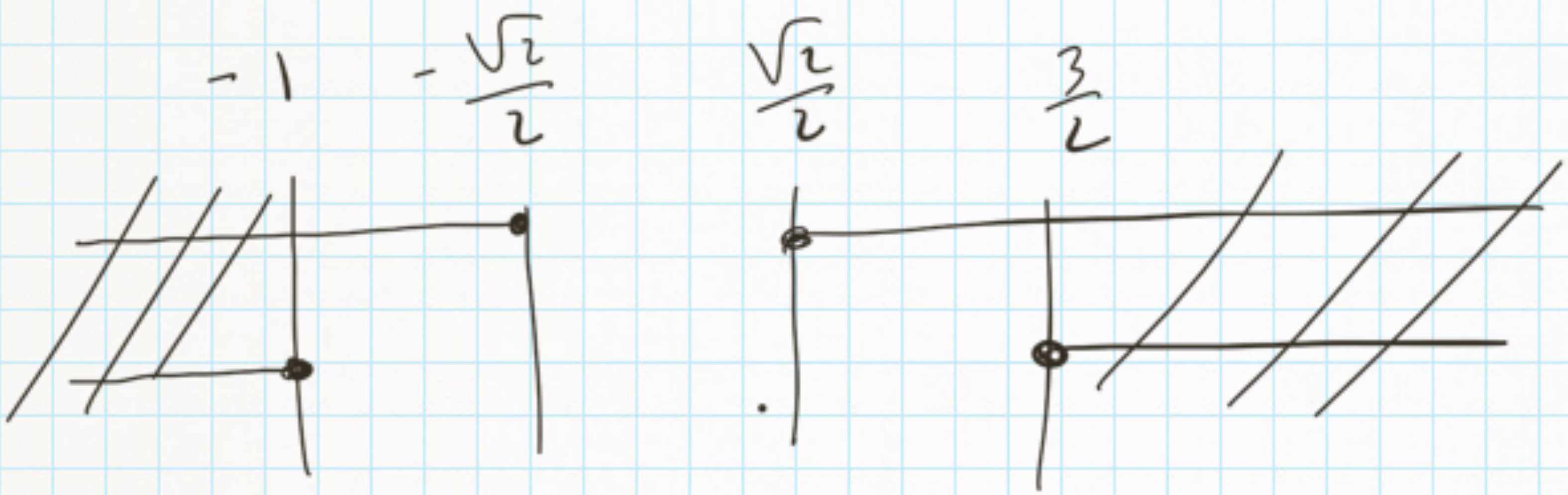
$$\Downarrow$$

$$\begin{cases} -\frac{\sqrt{2}}{2} < x < \frac{\sqrt{2}}{2} \\ \text{non ha soluzioni} \end{cases}$$

NESSUNA SOLUZIONE

Risolviamo

$$\begin{cases} 2x^2 - 1 \geq 0 \\ 2x^2 - x - 3 \geq 0 \end{cases} \qquad \begin{cases} x \leq -\frac{\sqrt{2}}{2} \ \vee \ x \geq \frac{\sqrt{2}}{2} \\ x \leq -1 \ \vee \ x \geq \frac{3}{2} \end{cases}$$



$$\left]-\infty, -1\right] \cup \left[\frac{3}{2}, +\infty\right[$$

ERRORE COMUNE

$$|2x^2-1| > x+2$$

$$\begin{cases} x \geq 0 \\ 2x^2-1 > x+2 \end{cases} \vee \begin{cases} x < 0 \\ 1-2x^2 > x+2 \end{cases}$$

Non si discute il segno della variabile x, ma dell'argomento del valore assoluto!

Si può usare un altro modo per risolvere la disequazione

$$|a(x)| \geq b(x) \iff a(x) \leq -b(x)$$

oppure $a(x) \geq b(x)$

$$\left| \frac{x+1}{x+3} \right| \geq x$$

① $\dfrac{x+1}{x+3} \leq -x$ oppure ① $\dfrac{x+1}{x+3} \geq x$

① $$\frac{x+1}{x+3} + x \leq 0 \Rightarrow \frac{x+1+x(x+3)}{x+3} \leq 0$$

$$\frac{x+1 \quad x^2+3x}{x+3} \le 0$$

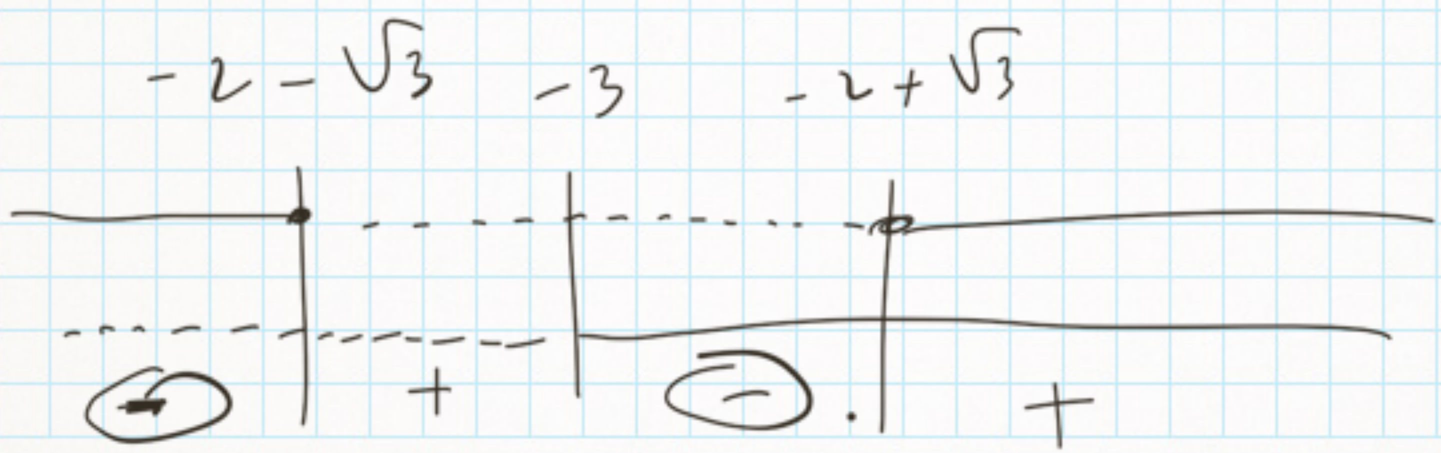
$$\boxed{\begin{aligned} 12 &= 3\cdot 4 \\ &= 3\cdot 2^2 \end{aligned}}$$

$$\frac{x^2+4x+1}{x+3} \le 0$$

$$x^2+4x+1 \ge 0 \iff \boxed{x \le -2-\sqrt{3} \ \vee \ x \ge -2+\sqrt{3}}$$

$$x_{1/2} = \frac{-4 \pm \sqrt{16-4}}{2} = \frac{-4 \pm \sqrt{12}}{2} = -2 \pm \sqrt{3}$$

$$x+3>0 \Rightarrow \boxed{x > -3}$$



Allora

$$\frac{x^2+4x+1}{x+3} \leq 0 \iff \boxed{x \leq -2-\sqrt{3} \ \vee \quad -3 < x \leq -2+\sqrt{3}}$$

Passiamo a risolvere la seconda disequazione

$$\frac{x+1}{x+3} \geq x$$

$$\frac{x+1}{x+3} - x \geq 0$$

$$\frac{x+1-x^2-3x}{x+3} \geq 0 \iff \frac{-x^2-2x+1}{x+3} \geq 0$$

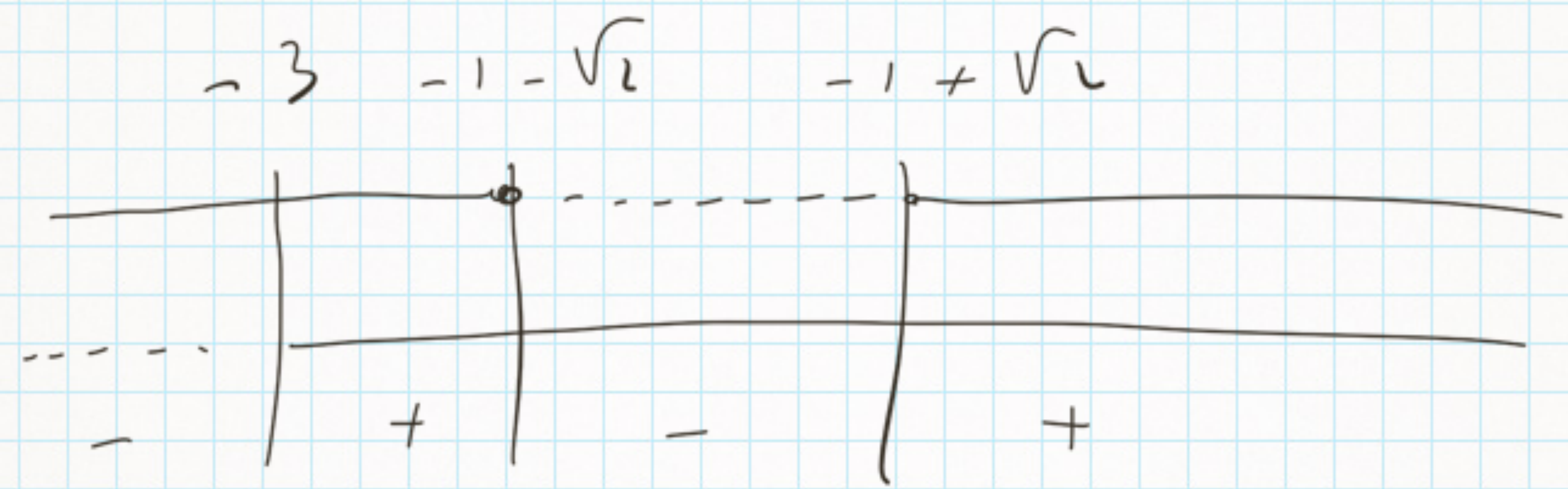
$$\iff \frac{x^2+2x-1}{x+3} \leq 0$$

$$x^2 + 2x - 1 \geq 0 \iff x \leq -1-\sqrt{2} \vee x \geq -1+\sqrt{2}$$

$$x^2 + 2x - 1 = 0$$

$$x_{1,2} = \frac{-2 \pm \sqrt{4+4}}{2} = \frac{-2 \pm \sqrt{8}}{2} = -1 \pm \sqrt{2}$$

$$x + 3 > 0 \Rightarrow x > -3$$

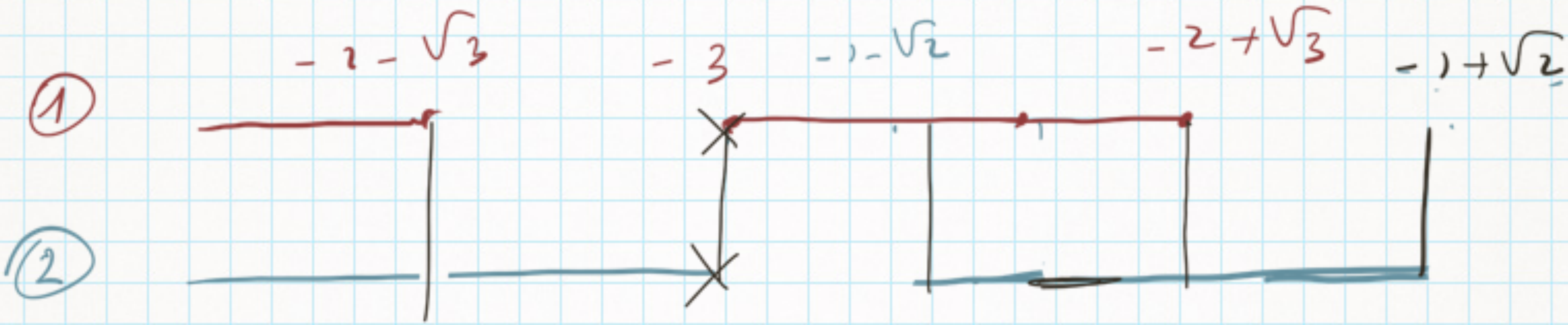


$$\boxed{x < -3 \quad \vee \quad -1-\sqrt{2} < x < -1+\sqrt{2}}$$

L'insieme delle soluzioni della disequazione

$$\left|\frac{x+1}{x+3}\right| > x$$

è l'unione dell'insieme delle soluzioni delle disequazioni risolte



Le soluzioni della disequazione sono

$$x \le -1+\sqrt{2}\,, \quad x \ne -3$$

$$\left|\frac{x^2-1}{x+2}\right| < x-2$$

DUE MODI :

(1)
$$\begin{cases} \dfrac{x^2-1}{x+2} \geq 0 \\[2ex] \dfrac{x^2-1}{x+2} < x+2 \end{cases} \quad \vee \quad \begin{cases} \dfrac{x^2-1}{x+2} < 0 \\[2ex] -\left(\dfrac{x^2-1}{x+2}\right) < x+2 \end{cases}$$

(2)
$$|a(x)| < b(x) \iff -b(x) < a(x) < b(x)$$

(conviene di più)

$$-(x-2) < \frac{x^2-1}{x+2} < (x-2) \iff \begin{cases} \dfrac{x^2-1}{x+2} < x-2 \\[2ex] \dfrac{x^2-1}{x+2} > -x+2 \end{cases}$$

Risolviamo la I disequazione

$$\frac{x^2-1}{x+2} < x-2$$

$$\frac{x^2-1}{x+2} - x + 2 < 0$$

$$(x+2)(-x+2) = 4 - x^2$$

$$\frac{x^2-1+(x+2)(-x+2)}{x+2} < 0$$

$$\frac{\cancel{x^2} - 1 + 4 - \cancel{x^2}}{x+2} < 0$$

$$\frac{3}{x+2} < 0 \iff x+2 < 0 \iff x < -2$$

$$\frac{x^2-1}{x+2} > -x+2$$

$$(x-2)(x+2) = x^2-4$$

$$\frac{x^2-1}{x+2} + x - 2 > 0$$

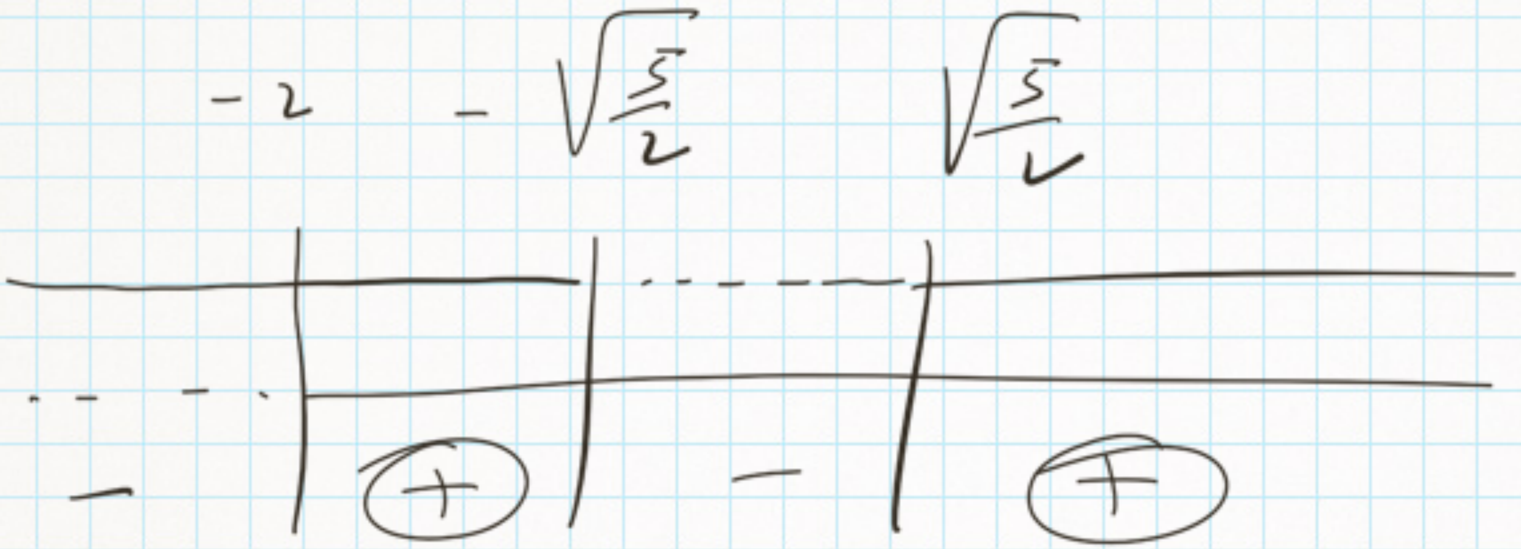
$$\frac{x^2-1+(x-2)(x+2)}{x+2} > 0$$

$$\frac{x^2-1+x^2-4}{x+2} > 0 \iff \frac{2x^2-5}{x+2} \geq 0$$

- $2x^2-5 > 0 \iff x < -\sqrt{\frac{5}{2}} \vee x > \sqrt{\frac{5}{2}}$

$2x^2-5=0 \iff x^2 = \frac{5}{2} \iff x = \pm\sqrt{\frac{5}{2}}$

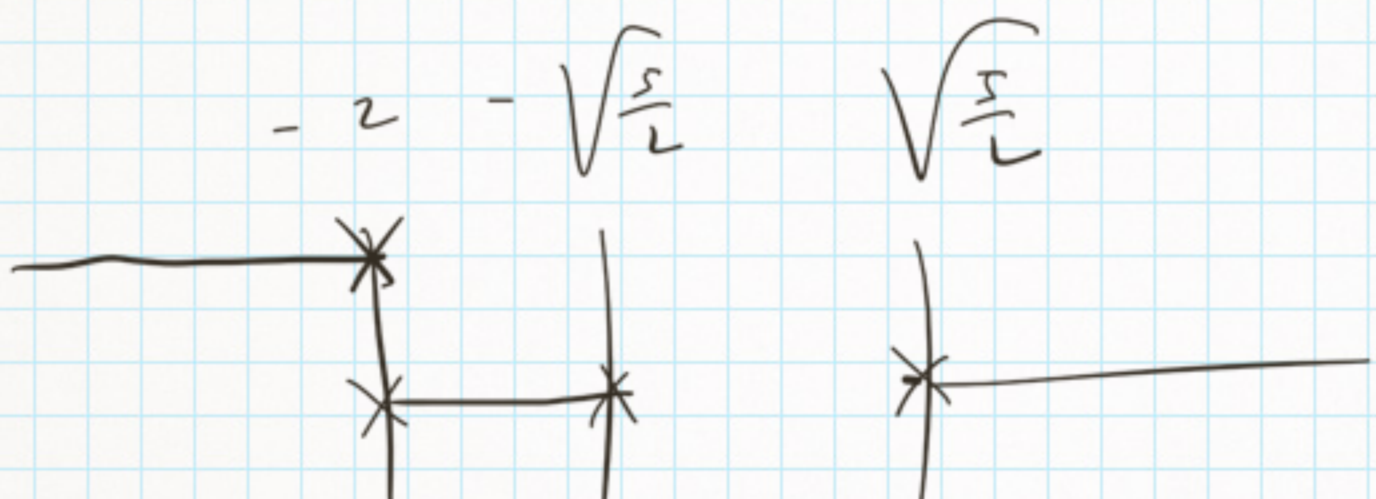
- $x+2>0 \quad x>-2$



$$-2 < x < -\sqrt{\frac{5}{2}} \quad \vee \quad x > \sqrt{\frac{5}{2}}$$

$$\begin{cases} x < -2 \\ -2 < x < -\sqrt{\frac{5}{2}} \quad \vee \quad x > \sqrt{\frac{5}{2}} \end{cases}$$

(SISTEMA)



SISTEMA IMPOSSIBILE !!

A CASA

$$\left| \frac{x^2-1}{x+3} \right| < 3$$

$$-3 < \frac{x^2-1}{x+3} < 3$$

SOLUZIONI : $\boxed{-2 < x < 5}$

Si trovino le soluzioni della disequazione

$$\frac{|2x-5|-2}{3-x} \geq 0$$

$$① \begin{cases} 2x-5 \geq 0 \\ \dfrac{2x-5-2}{3-x} \geq 0 \end{cases} \vee ② \begin{cases} 2x-5 < 0 \\ \dfrac{-2x+5-2}{3-x} \geq 0 \end{cases}$$

$$|2x-5| = \begin{cases} 2x-5 & 2x-5 \geq 0 \\ -(2x-5) & 2x-5 < 0 \end{cases}$$

Risolviamo il sistema

$$\begin{cases} 2x-5 \ge 0 \\ \dfrac{2x-5-2}{3-x} \ge 0 \end{cases} \qquad \begin{cases} x \ge \dfrac{5}{2} \\ \dfrac{2x-7}{3-x} \ge 0 \end{cases}$$

$$\begin{cases} x \ge \dfrac{5}{2} \\ \dfrac{2x-7}{x-3} \le 0 \end{cases}$$

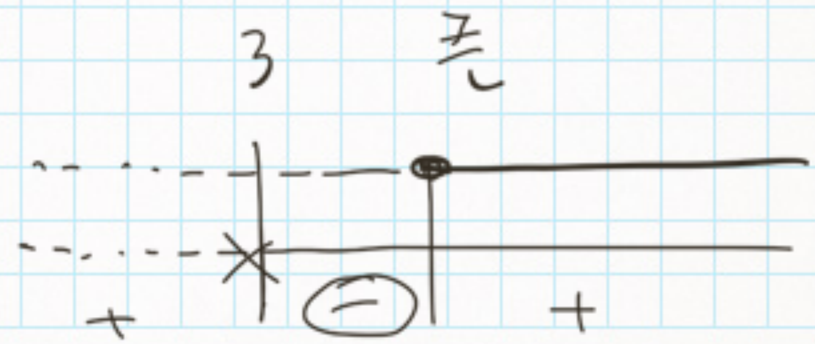
(HO MOLTIPLICATO PER -1)

$$\frac{2x-7}{x-3} \le 0$$

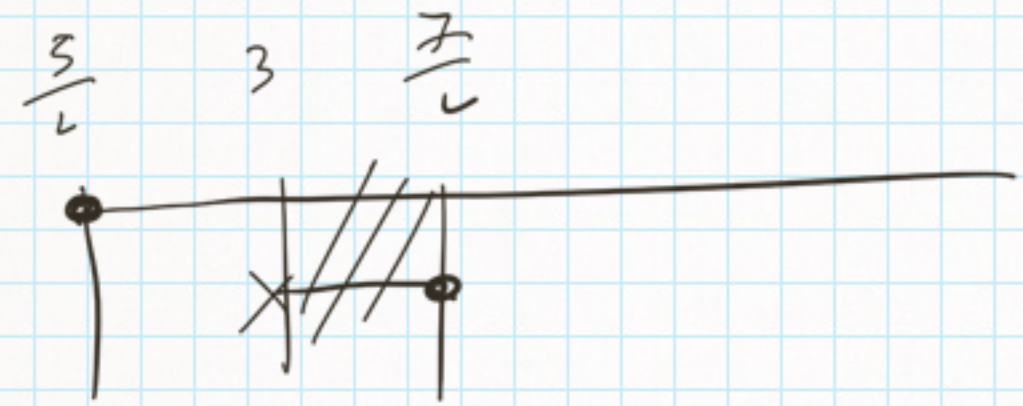
$2x-7 \ge 0 \Rightarrow x \ge \frac{7}{2}$

$x-3 > 0 \qquad x > 3$

$3 < x \le \frac{7}{2}$



$$\begin{cases} x \geq \frac{5}{2} \\ 3 < x < \frac{7}{2} \end{cases}$$



$$\boxed{3 < x \leq \frac{7}{2}}$$

Risolviamo il secondo sistema

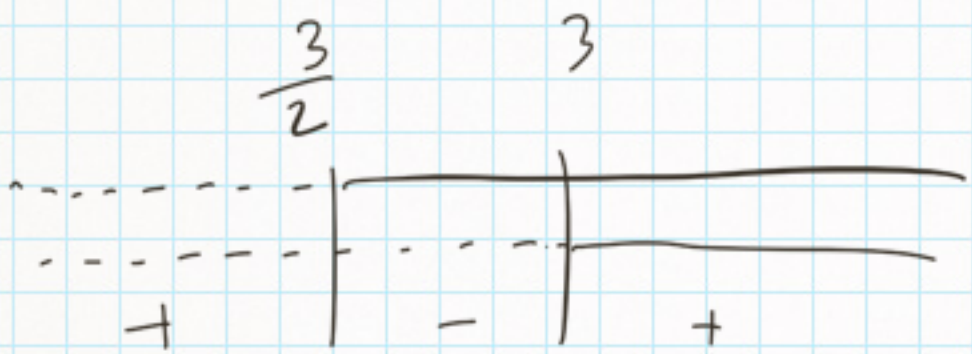
$$\begin{cases} 2x - 5 < 0 \\ \dfrac{5 - 2x - 2}{3 - x} \geq 0 \end{cases} \qquad \begin{cases} x < \frac{5}{2} \\ \dfrac{3 - 2x}{3 - x} \geq 0 \end{cases}$$

$$\frac{3-2x}{3-x} \geq 0 \quad (=) \quad \frac{-(2x-3)}{-(x-3)} \geq 0$$

$$(=) \quad \frac{2x-3}{x-3} \geq 0$$

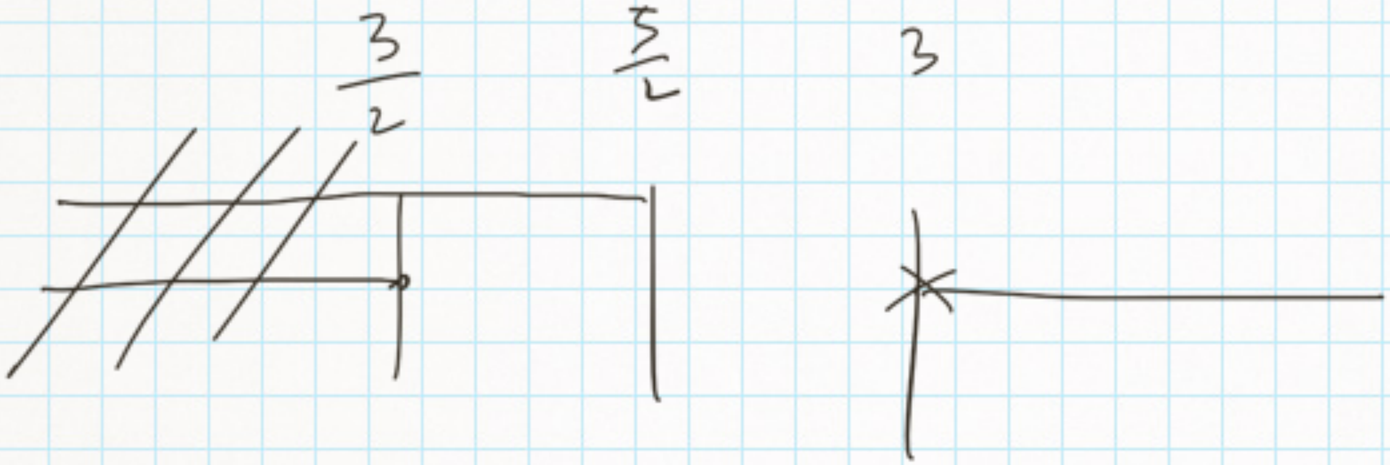
$2x-3 \geq 0 \qquad x \geq \frac{3}{2}$

$x-3 > 0 \qquad x > 3$



$x \leq \frac{3}{2} \vee x > 3$

$$\begin{cases} x < \frac{5}{2} \\ x < \frac{3}{2} \ \vee \ x > 3 \end{cases}$$



$$\boxed{x \leq \frac{3}{2}}$$

Concludendo l'equazione

$$\frac{|2x-5| - 2}{3-x} \geq 0$$

ha come sol: $\left]3, \frac{7}{2}\right] \cup \left]-\infty, \frac{3}{2}\right]$

$$\frac{2-4x}{|4-x|-2} \leq 0$$

$$① \begin{cases} 4-x \geq 0 \\ \dfrac{2-4x}{4-x-2} \leq 0 \end{cases} \vee ② \begin{cases} 4-x < 0 \\ \dfrac{2-4x}{x-4-2} \leq 0 \end{cases}$$

Risolviamo il sistema ①

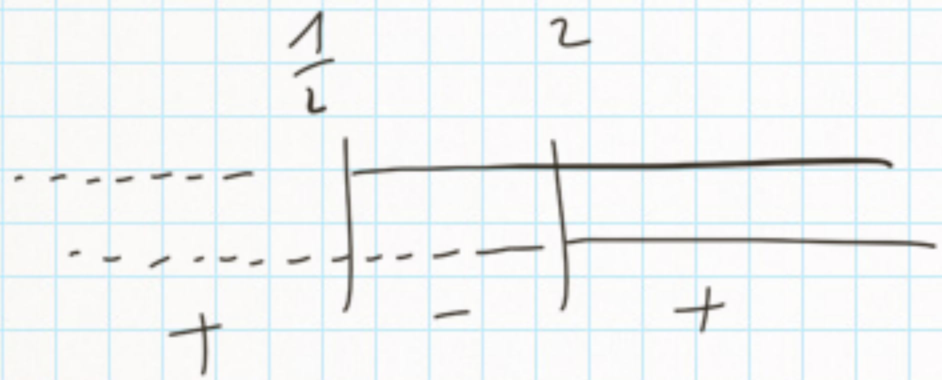
$$\begin{cases} x \leq 4 \\ \dfrac{2-4x}{-x+2} \leq 0 \end{cases} \qquad \begin{cases} x \leq 4 \\ \dfrac{-(4x-2)}{-(x-2)} \leq 0 \end{cases}$$

$$\begin{cases} x \leq 4. \\ \dfrac{4x-2}{x-2} \leq 0 \end{cases}$$

$$\frac{4x-2}{x-2} \le 0 \iff \frac{1}{2} < x < 2$$

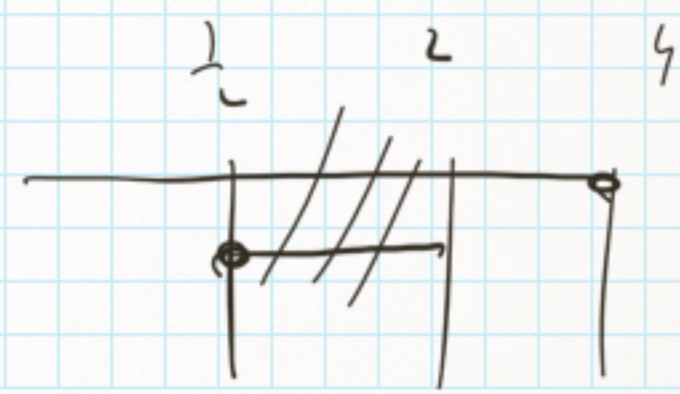
$$4x-2 \ge 0 \Rightarrow x \ge \frac{2}{4} = \frac{1}{2}$$

$$x-2 > 0 \qquad x > 2$$



Il sistema ha questa soluzione

$$\begin{cases} x \le 4 \\ \frac{1}{2} \le x < 2 \end{cases}$$



$$\boxed{\frac{1}{2} \le x < 2}$$

Risolvo il secondo sistema

$$\begin{cases} 4 - x < 0 \\ \dfrac{2-4x}{x-4-2} \geq 0 \end{cases} \qquad \begin{cases} x > 4 \\ \dfrac{2-4x}{x-6} \geq 0 \end{cases}$$

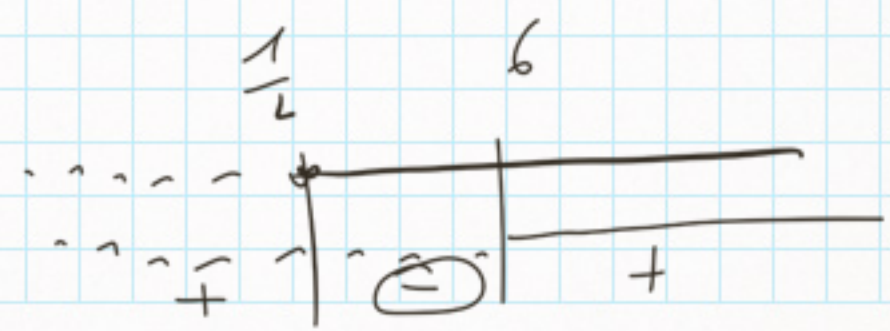
$$\begin{cases} x > 4 \\ \dfrac{4x-2}{x-6} \leq 0 \end{cases}$$ (moltiplico per -1)

RISOLVIAMO

$$\frac{4x-2}{x-6} \leq 0$$

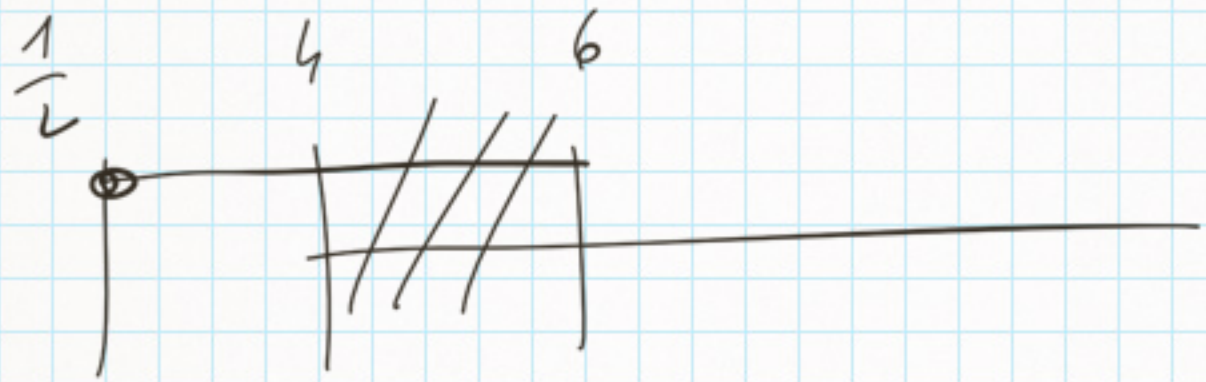
$4x - 2 \geq 0 \qquad x \geq \frac{1}{2}$

$x - 6 > 0 \qquad x > 6$



Il sistema ha le seguenti soluzioni

$$\begin{cases} x > 4 \\ \frac{1}{2} \le x < 6 \end{cases}$$



$$\boxed{4 < x < 6}$$

Consideriamo la disequazione

$$\frac{2-4x}{|4-x|-2} \le 0$$ ha per soluzioni

$$\left[\frac{1}{2}, 2\right[\; \cup \; \left]4, 6\right[$$

A CASA

$$\frac{|x-1|-5}{2x+1} \le 0$$

Soluzioni : $x \le -4 \vee -\frac{1}{2} < x \le 6$